

Arctic Sea Ice Exploration: An Example of a Mathematical Expedition in Polar Science

LINCS Science Discussion Group
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Adapted from the original exercise created by William Bauldry, Appalachian State University, and the Math Awareness Month 2013 version by Victor Donnay, Bryn Mawr College and Thomas Pfaff, Ithaca College.

Background

European explorers, beginning with Cabot's 1497 attempt to sail to the Orient from England, searched for the *Northwest Passage*, a route through the Arctic Ocean along the coast of Canada. The Norwegian explorer Amundsen was the first to complete the journey, though it took from 1903 to 1906. In 1957, the U.S. Coast Guard Cutter *Storis* became the first U.S. vessel to circumnavigate the North American continent, a 22,000 mile trek.

The problem is the Arctic Ocean is covered by a sea ice pack nearly all the time, and the passage is closed. Since the beginning of the Industrial Revolution, global temperature averages have risen overall causing more of the ice pack to melt in the summer, which leaves more ocean open. Melting sea ice provides an example of a *positive feedback loop* (also called non-linear feedback). Ice is very reflective giving the arctic region a high *albedo*; ice reflects up to 70% of the sun's energy. The ocean is darker, reflecting only 6% of the sun's energy, so as the ice pack retreats, the area's albedo gets lower. More energy is absorbed by ocean water than by sea ice thereby increasing the temperature, causing more ice to melt leading to more open water, creating a positive feedback loop.

The *National Snow and Ice Data Center* (NSIDC), part of the Cooperative Institute for Research in Environmental Sciences (CIRES) at the University of Colorado at Boulder, has collected data provided by satellites, over-flights, submarines, and other observations measuring the amount of sea ice in the Arctic Ocean for several decades. NSIDC's research and scientific data management activities are supported by NASA, the National Science Foundation (NSF), the National Oceanic and Atmospheric Administration (NOAA), and other federal agencies.



Figure 1: Arctic Sea Ice
Image courtesy of the National Snow and Ice Data Center, nsidc.org.

The amount of ice is at a minimum in September, the end of summer. Figure 2 shows the September average *extent* (the area covered by ice) of total Arctic sea ice in millions of square kilometers versus the year from 1979 to 2013.

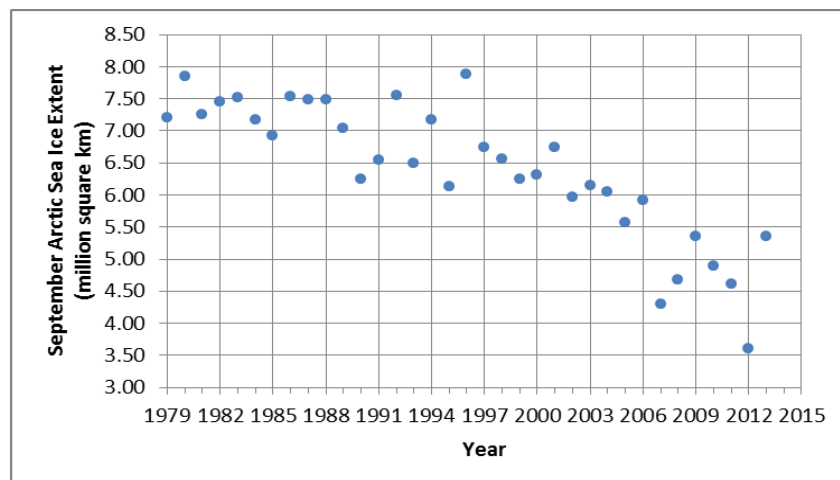


Figure 2: Average September Arctic Sea Ice Extent, 1979 to 2013

Part I: Class Work

1. From the data given in Figure 2, how much ice was in the Arctic in September 1988, September 1998 and September 2008? What are the units? What is the overall trend in the amount of sea ice during the 1979-2013 time period?
2. On Figure 2, draw a line that appears to fit the data as best as possible. From your drawing, calculate the slope of the line. What is the appropriate unit of measurement for slope? How does slope describe the rate at which the extent of sea ice is changing? How does your value of slope connect with the overall trend of the sea ice extent?
3. It is important to communicate the value of slope so that it is understandable to the average citizen: state the LOSS of sea ice for the next year using the slope you determined in #2. In the U.S., miles are a more familiar unit of distance than km. Convert your LOSS estimate to square miles, and then compare that value to a known region, such as the size of your state or county.
4. The annual decrease (sometimes called the trend) is often described in terms of percentage. Take the slope you calculated in (2) and convert it to a percentage. What are the advantages or disadvantages to this way of describing the annual decrease?
5. It is also interesting to analyze how the rate of change may itself be changing over time. Sketch (by hand) lines that best fit the data for the following time intervals: 1979-1994; 1994-2003; 2003-2012. What is the slope of each line? Over time, what does this say about how the slopes are changing? What does this mean in the context of the extent of Arctic sea ice?

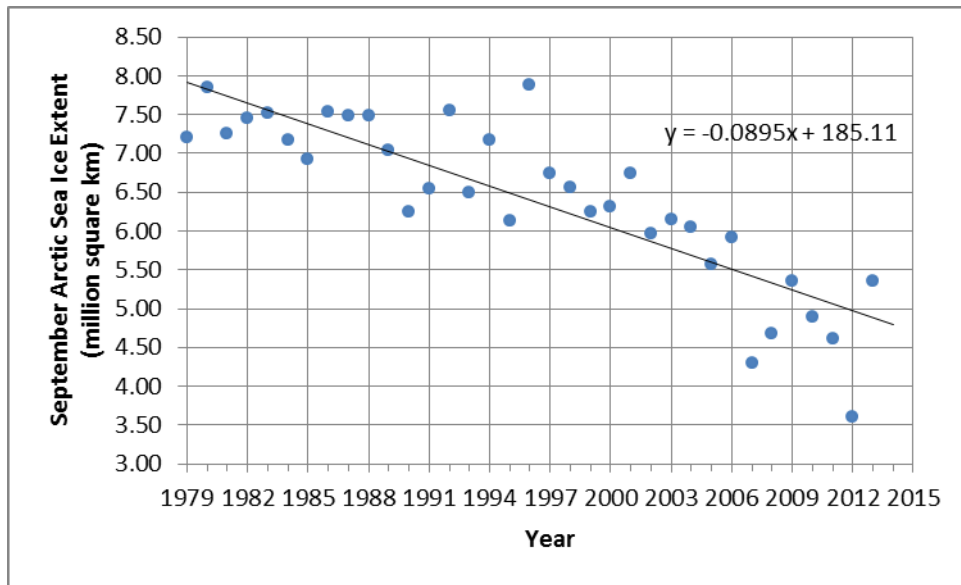


Figure 3: Average September Arctic Sea Ice Extent, 1979 to 2013, with Line of Best Fit

6. Figure 3 shows both the equation and the graph of the *line of best fit* as calculated in Excel with the Trendline command. The equation of the line of best fit is one *mathematical model* for average September Arctic sea ice extent, $y = -0.0895x + 185.11$. Use this linear model (linear equation) to calculate the amount of ice in September 2003 and in September 2012. How do these values compare to the actual data values?

7. If present trends continue, how much Arctic sea ice will there be in September 2020? In what year does this model predict that the Arctic will be ice free in September? Looking at the data, do you think this prediction is accurate? Why or why not?

8. What are the pros and cons of having an open Arctic passage?

MONTHLY ARCTIC SEA ICE EXTENT AVERAGES (million square km)

Year	1979	1980	1981	1982	1983	1984	1985	1986	1987
May	14.06	14.04	13.90	14.17	13.54	13.68	14.23	13.52	13.81
June	12.59	12.31	12.57	12.69	12.35	12.20	12.40	12.10	12.57
July	10.47	10.39	10.62	10.75	10.91	10.15	10.09	10.47	9.98
August	8.15	8.04	7.86	8.26	8.36	7.87	7.46	8.01	7.69
September	7.20	7.85	7.25	7.45	7.52	7.17	6.93	7.54	7.48
October	9.39	9.46	9.19	9.98	9.64	8.84	8.88	9.89	9.29

Year	1988	1989	1990	1991	1992	1993	1994	1995	1996
May	13.69	12.98	13.30	13.51	13.25	13.54	13.73	13.04	13.06
June	12.02	12.31	11.68	12.23	12.13	11.99	12.10	11.55	12.10
July	10.04	10.38	9.62	9.68	10.61	9.66	10.22	9.15	10.36
August	7.90	7.92	6.82	7.40	7.86	7.29	7.61	6.68	8.17
September	7.49	7.04	6.24	6.55	7.55	6.50	7.18	6.13	7.88
October	9.47	9.52	9.35	9.16	9.60	9.18	9.48	8.94	9.39

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005
May	13.32	13.80	13.86	13.18	13.72	13.12	13.00	12.58	12.99
June	11.91	11.85	12.10	11.71	11.69	11.69	11.77	11.51	11.29
July	9.59	9.62	9.59	9.75	9.22	9.49	9.46	9.60	8.93
August	7.30	7.49	7.38	7.21	7.47	6.53	6.85	6.83	6.30
September	6.74	6.56	6.24	6.32	6.75	5.96	6.15	6.05	5.57
October	8.76	8.85	9.10	8.92	8.59	8.81	8.65	8.48	8.45

Year	2006	2007	2008	2009	2010	2011	2012	2013	2014
May	12.62	12.89	13.16	13.39	13.10	12.79	13.13	13.10	
June	11.06	11.49	11.46	11.49	10.87	11.01	10.97	11.58	
July	8.67	8.13	9.06	8.82	8.39	7.92	7.94	8.45	
August	6.52	5.36	6.06	6.26	5.98	5.52	4.72	6.09	
September	5.92	4.30	4.68	5.36	4.90	4.61	3.61	5.35	
October	8.33	6.77	8.42	7.52	7.71	7.10	7.00	8.10	

Table 1: Monthly Arctic Sea Ice Extent Averages (million square km)
 National Snow and Ice Data Center, Univ. of Colorado, Boulder; <http://nsidc.org/arcticseaicenews/>

Part 2: Group Work

1. List the month your group is assigned: _____.
2. Use NSIDC's *Arctic Sea Ice Extent Averages* data given in Table 1 for your assigned month to derive a linear function giving the overall trend of the average sea ice extent.
3. Plot the linear function and your data on the same graph.
4. Predict your month's 2014 value. Use your model to predict when the Arctic will be ice free in your month.
5. Explain why the slope of the linear function describes the rate of change of Arctic sea ice extent. What is the trend as a percentage?
6. Is the trend you found reasonable? Why or why not? How does it differ from the September trend?

Suppose your group has been asked to present to a community group the results of your analysis.

Your report must include:

1. Your project team members' names.
2. Your linear model and predictions for 2014 with a justification of its appropriateness.
3. A graph showing your model with the data points.
4. A discussion of the trend you calculated and whether or not it reasonably describes the data.

You may produce your final report as your choice of

- _ a standard paper (in pdf, doc, docx, ...)
- _ a slide-show (Impress, Keynote, Powerpoint, or pdf)
- _ a video (in Quicktime or Windows Media format; mp4, mpeg, mov, wmv, or avi; not flv or swf)

Extra Credit Option: Explore a quadratic or other non-linear model of the data.

Lesson Plan Notes

This lesson can be taught in a variety of ways ranging from a teacher centered presentation to having students working through the assignment on their own or in groups and then presenting their findings to the class.

0. Lesson launch. Engage students in a discussion of Arctic sea ice. What do students know about this? What questions do they have? Show some pictures or a short video about the issue (for example, the video *A New Climate State: Arctic Sea Ice 2012*, <http://www.youtube.com/watch?v=ZYaubXBfVqo>) One might show some of the video at the start of class as an introduction and more at the end as a wrap up.
1. The Graph. Understanding and paying attention to the units is important. Often in math problems, there are no units given, consequently students do not always pay attention when there are units. The horizontal axis measures time in years. The vertical axis gives the average extent of Arctic sea ice in September. The units are square kilometers measured in millions (million km²). When answering the questions in the lesson, the students should always include the units.
2. Finding the line of best fit. When doing this by hand, there will be a lot of variation in the answers. As long as the students come up with something reasonable, that is fine. One can have an animated discussion with students explaining why they drew the line they did and seeing the variation among the results. This would provide a teachable moment for the need to have a precise definition of line of best fit.
3. Many students are not used to dealing with slope in the context of units and could find this question challenging. But using units helps in connecting slope to rate of change. WolframAlpha: Computational Knowledge Engine (<https://www.wolframalpha.com/>) is a good resource to come up with interesting comparisons. For example, a student can simply type in something like “size of Virginia” and it will return all sorts of data about the area (and elevation, etc.) in a variety of units. *You may find that this site has the potential to generate many, many interesting math and science questions!*
4. Have a discussion of slope with units as describing rate of change. Then look at how could describe loss as a percentage, which are *relative* (no units).
5. Examine the change in the slopes. Lead the discussion to consider the rate of change of the slope. (The rate of change of the rate of change.) What would be the units of measurement?

The data in the last three years are all below the line of best fit, with the most of the recent values being significantly below the line. So the best fit line could be a poor predictor of the future because it weights earlier years equally with recent years. But the recent years suggest that the behavior of the sea ice is changing. The positive feedback loop might be accelerating the melting. The problem of sea ice melt is very complex due to the non-linear feedbacks involved; the scientific community has not reached consensus on when they think the ice free stage will arrive.

Possible extension: Draw a curve that seems to best fit the data from 1979 – 2013. What is the concavity of the curve? Compare to the lines drawn. Lead a discussion of how a concave down curve has lines of decreasing slope. Might want to discuss “increasing” and “decreasing” in terms of slope and then in terms of change in slope.

Possible extension: Respond to the following statement: *If positive feedback causes the rate of change to be increasing, a linear trend will underestimate sea ice loss.*

6. In Excel, we used the Chart – Add Trendline feature and found that the line of best fit is given by $y = -0.0895x + 185.11$.

In 2003, $y = -0.0895(2003) + 185.11 = 5.84$ million km^2

And in 2012, $y = -0.0895(2012) + 185.11 = 5.04$ million km^2

7. Using the model, students need to find y when $x = 2020$:

$y = -0.0895(2020) + 185.11 = 4.32$ million km^2 .

To determine when the Arctic will be ice free, the students must solve the equation:

$-0.0895x + 185.11 = 0$ which gives $x = 2068$.

Possible extension: Using the data for just the past several years (ex. 2009-2013), predict what the extent of Arctic sea ice will be in 2014 and predict when the Arctic will be ice free in September. Using just the more recent data will lead to a significantly shorter estimate.

Although this model was written in x - y notation, it perhaps would be better to choose independent and dependent variables that better connect to the physical quantities and units, such as t for time in years and A for area (extent) of Arctic sea ice in million square kilometers.

8. Some pros of having an ice free passage through the Arctic sea: it will make shipping through the Northwest Passage possible, which will reduce cost and travel time for goods. It might also give access to more natural resources. Cons include that will make it difficult for polar bears to survive. And it is an indicator that serious climate changes are taking place on the earth and these changes will have far reaching consequences.

Possible extension: have each student choose a different country and do a bit of research to see what that country's interests and concerns might be regarding an open passage. Countries to include: the United States, Canada, China, Russia, Greenland, Mexico, Australia, Germany, Japan, Chile, South Africa. (Of course, include others depending on the size and interests of the class.) Hold a Model United Nations style debate, including discussions of mining, oil exploration, tourism, defense, etc.

A sampling of resources for teachers and students:

A New Climate State: Arctic Sea Ice 2012 <http://www.youtube.com/watch?v=ZYaubXBfvqo>

National Snow and Ice Data Center www.nsidc.org

Animations by Andy Lee Robinson:

Arctic Death Spiral animation (June 13, 2013) <http://youtu.be/bL7D5eRIDxk>

Arctic Sea Ice Death Spiral animation (February 21, 2013) <http://youtu.be/qUO23Y179pU>

Arctic Sea ice Minimum Volumes 1979-2012 (April 22, 2013) <http://youtu.be/YgiMBxaL19M>

Arctic Sea Ice Collapse 1979-2012 (PIOMAS) (August 21, 2013) <http://youtu.be/oFMDmS783Ro>

Five Myths about Arctic Sea Ice: Brendan P. Kelly (March 25, 2013)

Video published by AGU: <http://www.youtube.com/watch?v=EkXXPKnlmg&list=PL1D498B1B2807630E>

Document: <http://sites.agu.org/sharingscience/files/2013/03/Myths-About-Sea-Ice.pdf>

Arctic sea ice just hit a record low. Here's why it matters. By Brad Plumer, 8/28/2012, WonkBlog

<http://www.washingtonpost.com/blogs/wonkblog/wp/2012/08/28/arctic-sea-ice-just-hit-a-record-low-heres-why-it-matters/>